EES351 2021/1 Part II.4 Dr.Prapun

5 Angle Modulation: FM and PM

5.1. We mentioned in 4.2 that a sinusoidal carrier signal

$$A\cos(2\pi f_c t + \phi)$$

has three basic parameters: amplitude, frequency, and phase. Varying these parameters in proportion to the baseband signal results in amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM), respectively.

5.2. As in 4.63, we will again assume that the baseband signal $m(t)$ is

(a) band-limited to B; that is, $|M(f)| = 0$ for $|f| > B$

and

(b) bounded between $-m_p$ and m_p; that is, $|m(t)| \le m_p$.

5.1 PM and Introduction to FM

Definition 5.3. ***Phase modulation*** (***PM***):

$$x_{\text{PM}}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$$

- max phase deviation:

Definition 5.4. The main characteristic[22] of ***frequency modulation*** (**FM**) is that the carrier frequency $f(t)$ would be varied with time so that

$$f(t) = f_c + k_f m(t), \tag{72}$$

where k_f is an arbitrary constant.

- The subscript "$_f$" in k_f is there to distinguish the constant from a similar constant in PM.
- $f(t)$ is varied from $f_c - k_f m_p$ to $f_c + k_f m_p$.
- f_c is assumed to be large enough such that $f(t) \geq 0$.

Example 5.5. Figure 34 illustrates the outputs of PM and FM modulators when the message is a unit-step function.

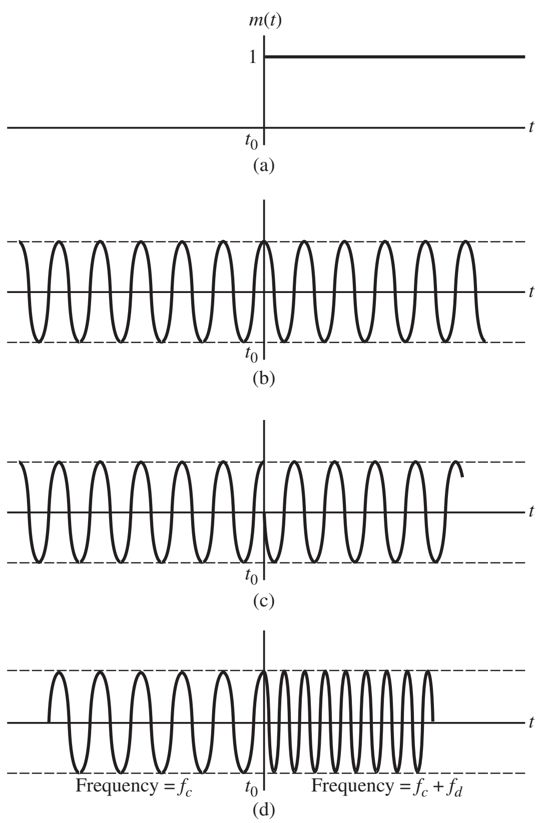


Figure 34: Comparison of PM and FM modulator outputs for a unit-step input. (a) Message signal. (b) Unmodulated carrier. (c) Phase modulator output (d) Frequency modulator output. [15, Fig 4.1 p 158]

[22]Treat this as a practical definition. The more rigorous definition will be provided in 5.16.

- For the PM modulator output,
  - the (instantaneous) frequency is f_c for both $t < t_0$ and $t > t_0$
  - the phase of the unmodulated carrier is advanced by $k_p = \frac{\pi}{2}$ radians for $t > t_0$ giving rise to a signal that is discontinuous at $t = t_0$.
- For the FM modulator output,
  - the frequency is f_x for $t < t_0$, and the frequency is $f_c + f_d$ for $t > t_0$
  - the phase is, however, continuous at $t = t_0$.

Example 5.6. With a sinusoidal message signal in Figure 35a, the frequency deviation of the FM modulator output in Figure 35d is proportional to $m(t)$. Thus, the (instantaneous) frequency of the FM modulator output is maximum when $m(t)$ is maximum and minimum when $m(t)$ is minimum.

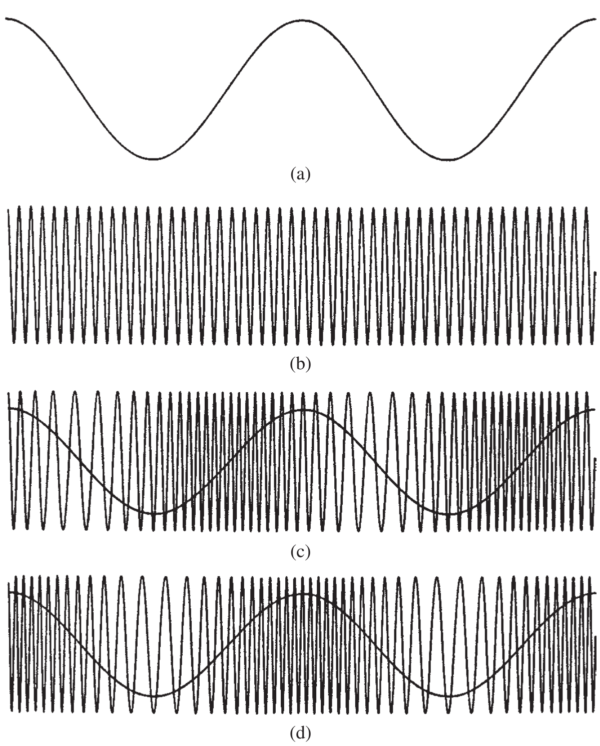


Figure 35: Different modulations of sinusoidal message signal. (a) Message signal. (b) Unmodulated carrier. (c) Output of phase modulator (d) Output of frequency modulator [15, Fig 4.2 p 159]

The phase deviation of the PM output is proportional to $m(t)$. However, because the phase is varied continuously, it is not straightforward (yet) to see how Figure 35c is related to $m(t)$. In Figure 39, we will come back to this example and re-analyze the PM output.

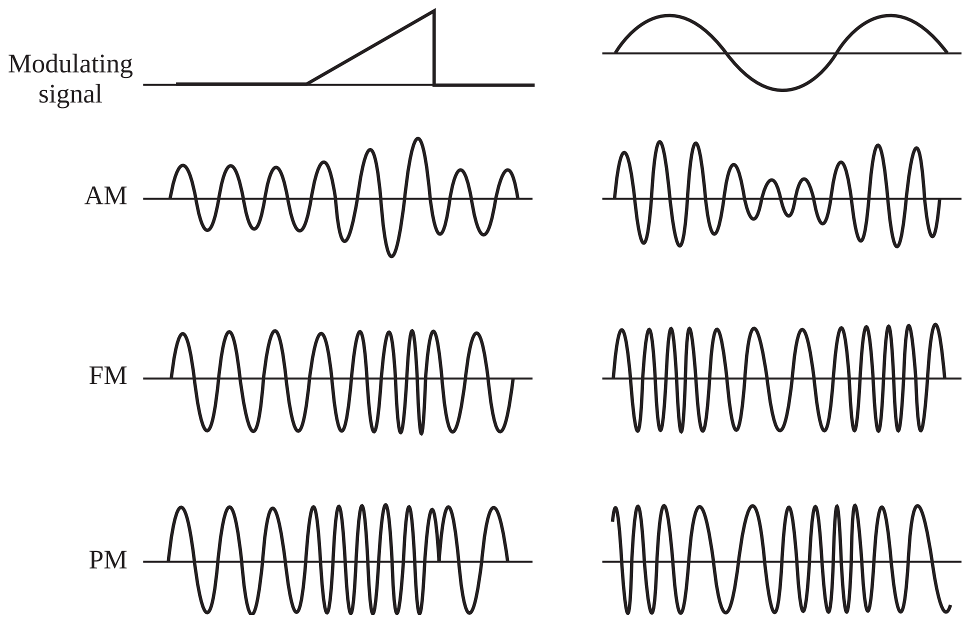


Figure 36: Illustrative AM, FM, and PM waveforms. [3, Fig 5.1-2 p 212]

Example 5.7. Figure 36 illustrates the outputs of AM, FM, and PM modulators when the message is a triangular (ramp) pulse.

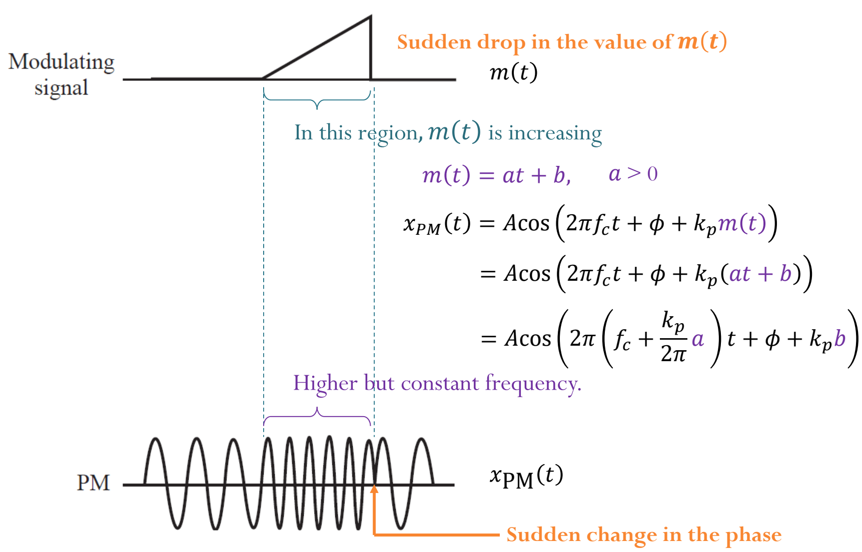
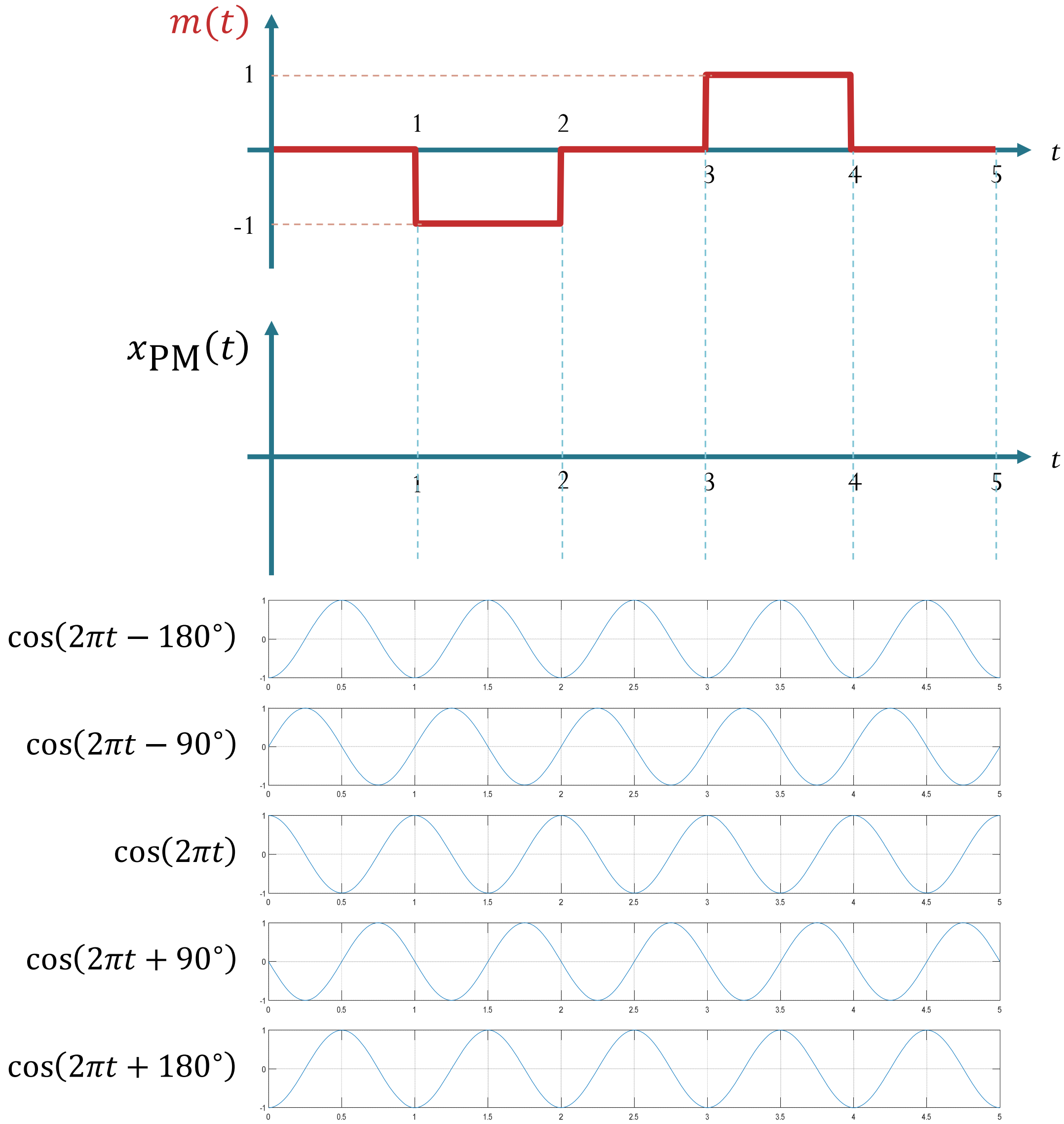


Figure 37: Explaining PM waveform in Figure 36.

Example 5.8. A PM signal is created from the message $m(t)$ by

$$x_{\text{PM}}(t) = 2\cos\left(2\pi f_c t + k_p m(t)\right).$$

Suppose $f_c = 1$ and $k_p = \frac{\pi}{2} = 90°$. For the message $m(t)$ plotted below. Plot the corresponding $x_{\text{PM}}(t)$.



See 5.20b and Example 5.21 for an alternative general method.

To understand more about FM, we will first need to know what it actually means to vary the frequency of a sinusoid.